RESEARCHING THE NATURAL RADIAL OSCILLATION FREQUENCIES AND CORRESPONDING OSCILLATION MODES OF THE CLADDING OF FUEL RODS OF THE VVER-1000 NUCLEAR REACTOR

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Introduction. All external influencing factors acting on the cladding of fuel rods are non-stationary with harmonic members of different frequencies. Such harmonic loadings can lead in some conditions to more impacting on the systems than the stationary loadings of same intensities, as well as they can induce the specific damaging impacts like the fretting [1]. Due to these circumstances, the problems about vibrations of the fuel rods in assemblies are of current interests, as well as the theme of this research, which deals with the natural oscillation frequencies and modes of the cladding of fuel rods of the VVER-1000 nuclear reactor. The purpose of this research is to develop the approach for evaluating the natural oscillation frequencies and modes on the base of finite differences technique for the fuel rod's cladding represented as the thick-walled cylinder, as well as to obtain the quantitative assessments of these natural oscillation frequencies and modes for fuel rod's cladding of the VVER-1000 nuclear reactor.

Mathematical model of the natural oscillations of the cladding. The average radius is about ten times greater than the thickness of the wall, and the length in the axial direction is more about four thousand times greater than the external radius for the claddings of fuel rods in the most of modern nuclear reactors, including the VVER-1000. Such thick-walled and long in axial direction cylinders can be considered using the hypotheses of the plane strain [2]:

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}+\omega^2\frac{\rho\left(1-\nu^2\right)}{E}u=0,\quad a<r<b\ , \tag{1}$$

$$\frac{du}{dr}+\frac{1}{r}u=0,\quad r=a, r=b\ , \tag{2}$$

where $u=u(r)$ is the function of the natural oscillation mode; r is the radial coordinate; ω is the natural oscillation frequency; ρ, E and ν are the density, the equivalent Young's modulus and the Poisson's ratio of the material of the cladding; a and b are the internal and external radii.

The equivalent Young's modulus and the Poisson's ratio in the equation (1) and the boundary conditions (2) can be represented considering with the hypotheses of the plane strain [2]:

$$E=E'/\left(1-\nu'^2\right),\ \ \nu'=\nu'/\left(1-\nu'\right), \tag{3}$$

where E' and ν' are the Young's modulus and the Poisson's ratio of the material of the cladding.

Using the (1), (2) to find the infinite number of natural oscillation frequencies and corresponded them the natural oscillation modes:

$$\omega_k,\ u^{(k)}(r),\quad k=1,2,\ldots, \tag{4}$$

The values and functions (4) are the most generalized characteristics of the dynamical properties of the cladding of fuel rods and it is necessary to find them further.

Finite differences for analysis of natural oscillations of the cladding. The finite differences technique is one of the most powerful approaches to analyse the natural oscillations [3]. The main idea of this approach is to consider the separate values of the function of the mode in some chosen points instead to consider this function as the continuous function [3]. The grid of the nodes is introduced on the researched domain, which in the case of the problem (1), (2) is reduced to the interval $a \le r \le b$, such as the coordinates of the grid's nodes are:

$$r_k = a + k\Delta r, \quad k = 0,1,2,\ldots,n,n+1, \tag{5}$$

where r_k is the coordinate of the grid's node with number k; $\Delta r = (b-a)/(n+1)$ is the step of the grid; n is the number of nodes inside the domain excluding the boundary points.

Than grid (5) had been defined, it is possible to introduce the nodal values u_k of the mode

$$u_k = u(r_k), \quad k = 0,1,2,\ldots,n,n+1, \tag{6}$$

Using well-known finite differences technique, it is possible to represent the differential equation (1) and the boundary conditions (2) in the terms of the nodal values (6):

$$\alpha_k u_{k-1} + \beta_k u_k + \gamma_k u_{k+1} = 0, \quad k = 1,2,\ldots n, \tag{7}$$

$$\alpha_0 u_0 + \beta_0 u_1 + \gamma_0 u_1 = 0, \quad \alpha_{n+1} u_{n+1} + \beta_{n+1} u_n + \gamma_{n+1} u_{n-1} = 0, \tag{8}$$

where $\alpha_0 = \nu/a - 3/(2\Delta r)$, $\beta_0 = 2/\Delta r$, $\gamma_0 = -1/(2\Delta r)$; $\alpha_k = 1/\Delta r^2 - 1/(2r_k\Delta r)$, $\beta_k = -2/\Delta r^2 + 1/r_k^2$, $\gamma_k = 1/\Delta r^2 + 1/(2r_k\Delta r)$; $\alpha_{n+1} = \nu/b + 3/(2\Delta r)$, $\beta_{n+1} = -2/\Delta r$, $\gamma_{n+1} = 1/(2\Delta r)$.

The equalities (8) are the discrete analogue of the differential equations (1), but equalities (8) are the discrete analogue of the boundary condition (2) in the point $r = a$ and $r = b$. The equalities (8) allows to exclude the nodal values u_0 and u_{n+1} from the equalities (7) corresponded to the value $k = 1$ and $k = n$. As the result, the problem (1), (2) can be reduced to the well-known in linear algebra eigenvalues and eigenvectors problem, which can be represented in matrix form:

$$([A] - \lambda[I]) \cdot \{u\} = \{0\}, \tag{9}$$

where $[A]$ and $[I]$ are some and unit matrices with size $n \times n$; $\lambda = -\omega^2\rho(1-\nu^2)/E$ is the eigenvalue of the matrix $[A]$; $\{u\} = (u_1 \quad u_2 \quad \ldots \quad u_n)^T$ is the eigenvector consisted of the nodal values of the natural oscillation mode; $\{0\}$ is the zero vector with size n.

The matrix $[A]$ is defined by the equalities (7), (8) and it is not presented here to exclude the cumbersome expressions. The eigenvalues λ can be found from the condition of existing of the nonzero solution of the homogeneous linear equations (10):

$$\det([A] - \lambda[I]) = 0. \tag{10}$$

Equation (10) allows to find the n eigenvalues λ, which approximately represent the first n natural oscillations frequencies (4). Substituting these eigenvalues into the linear equations (9) allow to the n homogeneous systems with nonzero solutions representing the corresponding eigenvectors and as result the natural oscillation modes. The number n must be sufficiently big to provide the required accuracy of first estimated natural oscillations frequencies and modes (4). To solve the equation (9) it is necessary to use the numerical methods, discussed in [4]. Due to the Hessenberg form of the matrix $[A]$, it is possible to use the procedure HQR2 [4], realising the QR-method for finding the eigenvalues and eigenvectors of this matrix.

Results for natural oscillation of the cladding of fuel rods and its discussing. The cladding of fuel rods used in the VVER-1000 nuclear reactors is made from the Zr-1%Nb alloy:

$$a = 3{,}9\text{mm},\quad b = 4{,}55\text{mm},\quad \rho = 6500\text{kg}/\text{m}^3,\quad E' = 96\text{GPa},\quad \nu' = 0{,}33. \qquad (11)$$

Some of computing results for the natural oscillations frequencies are presented in the Table 1.

Table 1 – The natural oscillations frequencies of the cladding of fuel rods used in the VVER-1000

Numbers of frequencies	Frequencies (Hz), computed using grids with different n		
	$n = 3$	$n = 10$	$n = 1000$
1	144773,94854173	145116,22162999	145192,80187767
2	3534603,34744291	3400744,43756552	3399949,64487786
3	7064831,64725440	6790021,61655265	6795171,60259326

From the table 1 we can see that the proposed approach based on the finite differences technique allows the excellent convergence of results with increasing the number of grid nodes. It is possible that the large values of the natural oscillations frequencies are due to neglecting the damping in the mathematical model (1), (2).

Conclusions. The obtained results allow to approve that the finite differences technique can be recommended to using in further researches for assessments the natural oscillations frequencies and modes of the cladding of fuel rods and other core's structures of nuclear reactors. The large values of the natural oscillations frequencies of the cladding of fuel rods used in the VVER-1000 reactors have evidenced about possibilities of the high-frequencies vibrations and it is necessary to research the impact of different kinds of damping on values of the natural oscillations frequencies.

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